Modeling Microwave Devices: A Symbolic Approach

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***Abstract*—A symbolic computation technique, which automatically converts an equivalent circuit model of an electron device into a set of closed-form S-parameter equations, is presented. It is shown how these equations are used to customize a computer-aided model generation process. Significant improvement has been observed in the computational efficiency of a modeling process utilizing these closed form equations.**

I. Introduction

One of the earliest symbolic programs was developed for mathematical integration [1]. A symbolic computation technique, which transforms an equivalent circuit model consisting of electrical components into a set of S-parameter equations, is presented. These equations are used to create a computer-aided modeling process customized for a specific microwave device.

Equivalent circuit models are commonly determined by optimization techniques which iteratively fit the parameters of a model to measured data of a device [2]. Measured two-port S-parameters are typically used in the modeling of microwave devices. In every iteration, a new set of model parameters, referred to as a solution in the following discussion, is generated and evaluated. The evaluation is done by using the new solution to compute the S-parameters of the model and comparing them with the measured ones. The solution is then accepted or rejected according to certain acceptance criteria.

Different optimization techniques are characterized by their unique ways of solution generation and acceptance criteria. Regardless of the optimization technique used, a large number of iterations is required to obtain a model. A little thinking should reveal that the S-parameter calculation, which has to be carried out repeatedly, consumes the major computational time of the modeling process. The calculation of S-parameters is the target for improvement in this letter.

Virtually all computer-aided modeling processes use a nodal analysis or its variations to compute scattering parameters (or, in general, network parameters) from a given equivalent circuit model [3]. Nodal equations are represented in the matrix form

$$\begin{bmatrix} Y_R & B \\ C & D \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} J \\ F \end{bmatrix}. \tag{1}$$

Partition Y_R is the nodal admittance matrix formed by considering only those components whose branch currents are not required to be determined. V is a vector of all node voltages. I contains the branch currents to be determined and the controlling current variables. The current contributions of the elements in I at each node are represented by partition B. J represents independent current sources and F contains independent voltage sources. The partitions C and D, when multiplied by V and I, respectively and equated to F, describe the branch constitutive equations of those components whose branch currents are used as controlling variables or are to be determined.

According to the nodal analysis technique, the coefficients of the simultaneous equations in (1) are numerically derived from the component values of the circuit model. Gaussian elimination and its modifications are commonly used to solve these equations for network parameters. This numerical calculation is adequate for the one-time operation in circuit simulation. However, the variations of model component values at each iteration of the modeling process require these equation coefficients to be updated and the network parameters resolved. The result is a lengthy circuit analysis process which has to be repeated for every new solution that has been generated.

A set of algebraic S-parameter equations in closed form can be manually derived and coded. The result is a special purpose modeling program that is customized for a specific device represented by the model. It is intuitively obvious that the manual process of deriving symbolic network parameter equations is tedious and error prone. Furthermore, equations have to be rederived every time the model topology is modified.

The next section explains the technique of symbolically transforming a model into a set of equations. These equations are then used in a parameter extraction program. A speed comparison between the conventional numerical approach and the new symbolic approach is performed and significant improvement has been observed.

II. Symbolic Approach

A symbolic computation process has been developed to generate a set of S-parameter equations in closed form. This set of equations is presented in a subroutine format $S_{ij} = f(P_k)$, where $S_{ij} (1 \leq i, j \leq 2)$ are the scattering parameters, $f(\bullet)$ represents a function, and P_k are the circuit elements of the model. This subroutine is compiled and linked with the control part of a modeling process to generate a customized parameter extraction program. The original numerical circuit analysis process is now replaced by a set of compiled equations. Since the S-parameters of a solution can be readily calculated by

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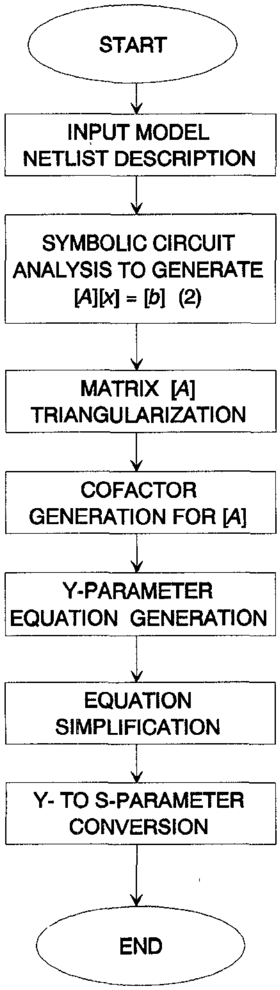

Fig. 1. The symbolic calculation process for scattering parameter equations.

numerically substituting new model parameters, significant speed gain can be obtained.

Fig. 1 shows the steps of this symbolic calculation process. A setup for generating Y-parameter equations for an HBT model is provided in Fig. 2 to demonstrate this process. The process first reads in the description of an equivalent circuit model. A symbolic matrix representation of the model in the format of (1) is then created. This is similar to traditional circuit analysis except that the matrix is written out symbolically. The symbolic matrix representation of the equivalent circuit in Fig. 2 is given in (2).

For simplicity of description, (2) is represented as $[A][x] = [b]$ in the following discussion. While all the unknown variables in vector $[x]$ can be found by multiplying the inverse matrix $[A]^{-1}$ with vector $[b]$, only i_1 and i_2 are required for the determination of scattering parameters. Now, refer to Fig. 2. According to the definition of Y parameters, $Y_{11} = i_1$ and $Y_{21} = i_2$ if V_i and V_o in vector $[b]$ are set to 1 and 0, respectively. Similarly, $Y_{12} = i_1$ and $Y_{22} = i_2$ if V_i and V_o in vector $[b]$ are set to 0 and 1, respectively. Observing that in both cases vector $[b]$ has a single nonzero element, the relationship between the Y-parameters and matrix $[A]$ can be summarized as follows:

$$\begin{aligned} Y_{11} = i_1 = \frac{\Delta(a_{n+1,n+1})}{|A|}, \text{ and } Y_{21} = i_2 = \frac{\Delta(a_{n+1,n+2})}{|A|}, \\ \text{when } (V_i = 1 \text{ and } V_o = 0), \\ Y_{12} = i_1 = \frac{\Delta(a_{n+2,n+1})}{|A|}, \text{ and } Y_{22} = i_2 = \frac{\Delta(a_{n+2,n+2})}{|A|}, \\ \text{when } (V_i = 0 \text{ and } V_o = 1), \end{aligned} \tag{3}$$

where $\Delta(a_{i,j})$ is the cofactor of the element in row i and column j of matrix $[A]$, n is the number of internal nodes of the model, and $|A|$ is the determinant of $[A]$.

In order to symbolically write out (3) into a subroutine, matrix $[A]$ is symbolically triangularized so that its determinant can be readily written as the product of its diagonal elements. A symbolic calculation such as the determination of cofactors and determinants commonly results in a very long expression in symbolic calculation. A simplification process is created to cancel out common factors in the denominator and numerator of the expression for any Y-parameter. The Y-parameters are then symbolically converted into the required S-parameters using the well-known relationship between them.

The subroutine containing the above symbolic S-parameter expressions is compiled into a customized parameter extraction program. Fig. 3 shows the execution times (measured in CPU seconds) of calculating scattering parameters using the conventional numerical approach and the new symbolic approach. This comparison shows an improvement of three to four times in the computational speed when the symbolic equations are used.

$$\left[\begin{array}{cccc|ccc} 1/(s^*L_B) & -1/(s^*L_B) & \cdots\cdots & 0 & -1 & 0 & 0 \\ -1/(s^*L_B) & -1/(s^*L_B) + s^*C_{be} + 1/R_B + s^*C_{bc} & \cdots\cdots & 0 & 0 & 0 & 0 \\ & & \ddots & & & \vdots & \\ & & \ddots & & & \vdots & \\ 0 & 0 & \cdots\cdots & 1/(s^*L_C) & & & 0 \\ \hline 1 & 0 & \cdots\cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots\cdots & 1 & 0 & 0 & 0 \\ 0 & 0 & \cdots\cdots & 0 & 0 & 0 & -1 \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_8 \\ \hline i_1 \\ i_2 \\ i_e \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ \hline V_i \\ V_o \\ 0 \end{array}\right] \tag{2}$$

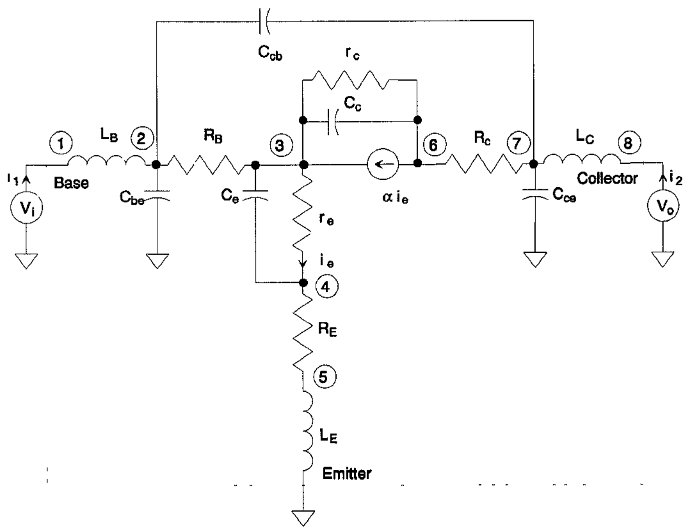

Fig. 2. Setup for determining the Y-parameters of an equivalent circuit model.

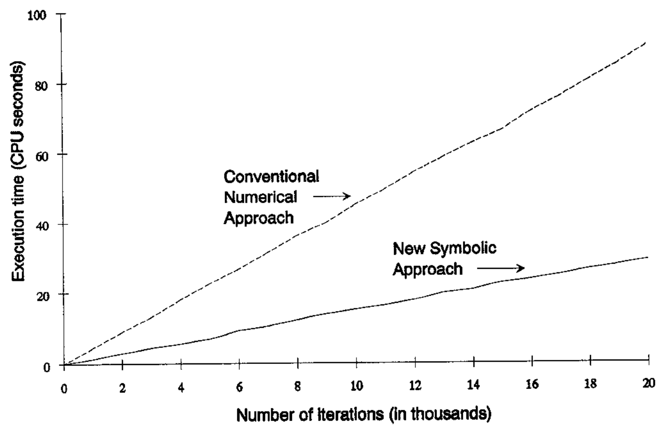

Fig. 3. Performance comparison of a numerical approach and a symbolic approach.

III. Summary

In summary, a symbolic calculation technique which creates a subroutine of closed form S-parameters from the topology of an equivalent circuit model is developed. The compiled model equations provide a significant speed advantage in the parameter extraction process. This technique can be readily modified to generate other network parameter equations by using the Y-parameters as an intermediate form. In addition, the application of this symbolic calculation is not limited to a modeling process, but can be adapted to any optimization process that uses network parameters to formulate an objective function.

References

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